**Echelon Form U and Row Reduced Form R**

* **Echelon matrix *U***
  + **The nonzero entries of *U*** have a echelon form.
  + **features:**
    - 1. **The pivots** are the first nonzero **entries** in their rows.
    - 2. Below each **pivot** is **a column** of the zeros, obtained by elimination.
    - 3. **Each pivot** lies to the right of the pivot in the row above.
    - 4. **Zero rows** come last.
  + For any ***m* by *n* matrix A**, there is **a permutation P**, **a lower triangular L** with unit diagonal, and **an *m* by *n* echelon matrix *U***, such that ***PA* = *LU***.
* **Row reduced form R**
  + make **pivots** **value** **1**, produce **zero** about the pivot
  + **MATLAB command: *R=rref(A)***
  + If ***A*** is invertible, *rref*(***A***) = ***I***.

**pivot variables and free variables and rank**

* **Pivot Variables:** correspond tocolumnswith pivots. (#: r)
* **Free Variables:** correspond tocolumnswithout pivots. (#: n-r)
* **special solution**
  + set one free variable = 1, others = 0.
  + solve ***Ax*** *= 0*, ***x*** is one **special solution**.
* **general solution to *Rx*=*0***
  + **the combinations** of special solutions form the nullspace -- **all solution to Ax=0**.
* **the rank of matrix**: = **the number** of pivot variables.
* **conclusion:**
  + If unknowns > equations (n>m), it has at least one special solution
  + **nullspace** has the same dimension as the number of free variables(special solutions).

**complete solution**

* ***x****complete =* ***x****particular +* ***x****nullspace*
  + ***x****particular* : comes from **solving** the equation with **all free variables** set to zero.
  + ***x****nullspace* : since they satisfy ***Ax*** = **0** ,
  + **all complete solutions** fill a plane which is parallel to the nullspace
* **steps**
  + **reduce form**: ***Ax = b*** to ***Ux = c*** to ***Rx = d***
  + **particular solution**: with set free variables = 0 ***(Rxp = d*** solves ***xp*** immediately***)***
  + **special solutions**(to ***Rx = 0***)
* **conclusion:**
  + **The last m-r rows of U and R** are **zero**, so there is a solution only if the last m-r entries of c and d are also zero.
  + There are **n-r special solutions** in the nullspace.
  + There are **m-r** **solvability comditions** on ***b*** or ***c*** or ***d***

**example**

* **elimination**
  + reveal **pivot variables(column space), free variables(nullspace), rank**
* 1. reduce [***A b***] to [***U c***], to reach a triangular system ***Ux*** *=* ***c***.
* 2. find the **condition** on the components of **b** to have a solution.
* 3. describe the **column space** of A: which plane in **Rm**?
* 4. describe the **nullspace** of A: which special solutions in **Rn**?
* 5. find **a particular solution** and the complete ***xp + xn***
* 6. reduce [***U c***] to [***R d***]
  + **special solutions** from ***R***
  + **particular solution** from ***d***